Enrollment No: _____     Exam Seat No:_____

# C.U.SHAH UNIVERSITY
## Winter Examination-2018

**Subject Name : Advanced Mathematics**

**Subject Code : 2TE02AMT3**                                    **Branch: All (Diploma)**

**Semester : 02          Date : 23/10/2018          Time : 02:30 To 05:30          Marks : 70**

Instructions:
  (1)  Use of Programmable calculator & any other electronic instrument is prohibited.
  (2)  Instructions written on main answer book are strictly to be obeyed.
  (3)  Draw neat diagrams and figures (if necessary) at right places.
  (4)  Assume suitable data if needed.

**Q-1**          **Attempt the following questions:**                                    **(14)**

**a)**  $\lim\limits_{x \to 0} \dfrac{a^x - 1}{x} = $ _____
(A) $\log_a e$   (B) $\log_e a$   (C) $e$   (D) 1

**b)**  $\lim\limits_{\theta \to 0} \dfrac{\sin m\theta}{\sin n\theta} = $ _____
(A) n/m   (B) m/n   (C) 1   (D) 0

**c)**  $\lim\limits_{x \to \sqrt{2}} \dfrac{x^2 + 3\sqrt{2}x - 8}{x^2 - 2} = $ _____
(A) 5   (B) 2   (C) 2/5   (D) 5/2

**d)**  $\dfrac{d(x^n)}{dx} = $ _____
(A) $x^{n-1}$   (B) $nx$   (C) $nx^n$   (D) $nx^{n-1}$

**e)**  $\dfrac{d(\log x)}{dx} = $ _____
(A) $-\dfrac{1}{x^2}$   (B) $\dfrac{1}{x}$   (C) $e$   (D) 1

**f)**  $\dfrac{d(\cot x)}{dx} = $ _____
(A) $\cos ec^2 x$   (B) $-\cos ec^2 x$   (C) $-\sec^2 x$   (D) $\sec^2 x$

**g)**  $\dfrac{d(\sin^{-1} x)}{dx} = $ _____
(A) $\dfrac{-1}{\sqrt{1-x^2}}$   (B) $\dfrac{1}{\sqrt{1-x^2}}$   (C) $\dfrac{1}{1+x^2}$   (D) $\dfrac{-1}{1+x^2}$

**h)**  $\int \dfrac{1}{x}\, dx = $ _____

(A) $-\dfrac{1}{x^2}+c$  (B) $\dfrac{1}{x^2}+c$  (C) $e^x+c$  (D) $\log x+c$

**i)** $\displaystyle\int \cos x\, dx = $ _____

(A) $-\sin x+c$  (B) $\sin x+c$  (C) $\sec x+c$  (D) $\cot x+c$

**j)** $\displaystyle\int_{2}^{5} x^3\, dx = $ _____

(A) $\dfrac{641}{4}$  (B) $\dfrac{609}{4}$  (C) $\dfrac{690}{4}$  (D) $\dfrac{614}{4}$

**k)** If $x=(3,2,1)$ and $y=(1,5,6)$ then $2x+3y=$ _____.

(A) $(9,19,20)$  (B) $(19,9,20)$  (C) $(20,19,9)$  (D) $(20,9,19)$

**l)** Magnitude of $3i-4j-2k=$ _____ .

(A) $\sqrt{14}$  (B) $\sqrt{7}$  (C) $\sqrt{29}$  (D) $\sqrt{13}$

**m)** $(2,-1,3)\times(-4,2,-6)=$ _____ .

(A) $(-8,-2,-18)$  (B) $(8,2,18)$  (C) $(0,0,0)$  (D) None of these

**n)** $(i+2j+k)\cdot(3k-2j+4i)=$ _____ .

(A) 3  (B) 5  (C) 7  (D) 11

## Attempt any four questions from Q-2 to Q-8

**Q-2** **Attempt all questions** (14)

**a)** If $\bar{a}=3i-j-4k$, $\bar{b}=-2i+4j-3k$ and $\bar{c}=-i+2j-5k$ then find $\left|\bar{a}+2\bar{b}-\bar{c}\right|$. (5)

**b)** Find unit vector which is perpendicular to $\bar{a}=5i+7j-2k$ and $\bar{b}=j-2k+3i$. (5)

**c)** Evaluate: $\displaystyle\lim_{n\to\infty}4\left[\dfrac{1^3+2^3+3^3+\ldots\ldots+n^3}{n^4}\right]$ (4)

**Q-3** **Attempt all questions** (14)

**a)** Prove that $\displaystyle\lim_{x\to\infty}\left[\sqrt{x}\left(\sqrt{x+p}-\sqrt{x}\right)\right]=\dfrac{p}{2}$ (5)

**b)** Evaluate: $\displaystyle\lim_{\theta\to0}\dfrac{\cos ec\theta-\cot\theta}{\theta}$ (5)

**c)** Find $\dfrac{dy}{dx}$ if $y=\dfrac{x^2-1}{x^2+1}$. (4)

**Q-4** **Attempt all questions** (14)

**a)** The equation of motion of a particle is $S=2t^3+3t^2-12t+5$. (5)
(i) Find velocity at $t=0$. (ii) Find acceleration at $t=1$.

**b)** Find $\dfrac{dy}{dx}$ if $y=\log\sqrt{\dfrac{1+\sin x}{1-\sin x}}$. (5)

**c)** If $\bar{a}=(2,-3,-1)$ and $\bar{b}=(1,4,-3)$ then find $\left(\bar{a}+\bar{b}\right)\times\left(\bar{a}-\bar{b}\right)$. (4)

**Q-5** **Attempt all questions** (14)

**a)** Prove that angle between two vectors $i + 2j$ and $i + j + 3k$ is $sin^{-1}\left(\sqrt{\dfrac{46}{55}}\right)$. **(5)**

**b)** Find $\dfrac{dy}{dx}$ if $e^x + e^y = e^{x+y}$. **(5)**

**c)** Evaluate: $\lim\limits_{x \to 0} \dfrac{3^{2x} - 2^{2x}}{x}$ **(4)**

**Q-6**          **Attempt all questions**                                                 **(14)**

**a)** Constant forces $3i - j + 2k$ and $i + 3j - k$ act on a particle and the particle moves from the point $2i + 3j + k$ to the point $5i + 2j + 3k$. Find the work done by the forces. **(5)**

**b)** Prove that $\displaystyle\int_{0}^{\frac{\pi}{2}} \dfrac{\sec x}{\sec x + \cos ecx} \, dx = \dfrac{\pi}{4}$. **(5)**

**c)** Find derivative of $y = \sqrt{x}$ using first principle. **(4)**

**Q-7**          **Attempt all questions**                                                 **(14)**

**a)** Evaluate: $\displaystyle\int \left[ \sqrt{1 + \sin 2x} + \sqrt{\dfrac{1 + \cos 2x}{1 - \cos 2x}} \right] dx$ **(5)**

**b)** Find the area of the region bounded between curve $y^2 = 4x$ and line $x = 2$. **(5)**

**c)** For what value of $p$, vectors $2i + 3j - k$ and $pi - j + 3k$ are perpendicular to each other? **(4)**

**Q-8**          **Attempt all questions**                                                 **(14)**

**a)** Evaluate: $\displaystyle\int x\sqrt{x^2 - a^2} \, dx$ **(5)**

**b)** If $y = e^x \sin x$ then prove that $\dfrac{d^2 y}{dx^2} - 2\dfrac{dy}{dx} + 2y = 0$. **(5)**

**c)** Evaluate: $\displaystyle\int_{0}^{10} W \, dx$   *Where* $W = \dfrac{3}{4} x \left( 1 + \dfrac{x}{10} \right)$ **(4)**